

How to Observe the Interference Effects of Top Quark Polarizations at Tevatron *?

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Using a simple analytic expression for $q\bar{q}, gg \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow b\bar{l}\nu_l\bar{b}'l'\nu_{l'}$ with the interference effects due to the polarizations of the t and \bar{t} , we demonstrate how the effects can be measured at Tevatron at 3σ level.

With the upgrade of Tevatron and the improvements of its two collider detectors, the data related to the production of the top quark are going to accumulate very quickly. Since top quark decays very rapidly after its production, it is expected that the spin information of the top quarks is preserved in the decay process. For lighter quark, this spin information is always smeared by the hadronization effect. Therefore, if such effect can be observed it will be the first time that one can observe the spin of a bare quark directly. However, it is challenging to investigate how one can determine the various detail properties of top quark in the complex hadronic collider environment.

Here we report our investigate an interesting physical consequence of the fact that the top quarks that are produced and decayed are supposed to be spin 1/2 particles^{1,2}. One of the important effect of the polarizations of unstable particles are that the different polarized intermediate states can interfere. In this sense, the observation of interference effects provides a unique possibility of direct observation of the spin of a quark.

We use a simple analytic result¹ for the differential cross section of $q\bar{q}$ and $gg \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow b\bar{l}\nu_l\bar{b}'l'\nu_{l'}$ based on an analytic helicity technique developed in ref. ³. The decays of W bosons and top quarks are taken into account in the narrow width approximation. The contribution due to off-shell top quarks or off-shell W bosons are negligible. The interference effects discussed here was also considered before in ref. ⁴, however, it was studied only numerically and was done in a rougher approximation. The analytic expressions obtained here can also be easily adapted to the leptonic colliders⁵. We shall also demonstrate that such interference effect can be detected at 3σ level at Tevatron with Main Injector if one uses proper observables².

The polarization density matrix for the process can be split into two main sections and written as

$$P = \frac{1}{N_s} \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda'_3, \lambda_4, \lambda'_4} P(i_{\lambda_1 \lambda_1} \bar{i}_{\lambda_2 \lambda_2} \rightarrow t_{\lambda_3 \lambda'_3} \bar{t}_{\lambda_4 \lambda'_4}) |\Pi_t(r_1)|^2 |\Pi_{\bar{t}}(r_2)|^2 \times P(t_{\lambda_3 \lambda'_3} \bar{t}_{\lambda_4 \lambda'_4} \rightarrow b\bar{b}l^+l^-\nu\bar{\nu}). \quad (1)$$

Here the initial the final polarizations are summed over. $N_s = 4(N^2 - 1)^2$ for gg initial states; and $N_s = 4N^2$ for $q\bar{q}$ initial states. $\Pi_q(r_i) = -i(r_i^2 - m_q^2)^{-1}$ represents the polarization independent components of the top quark propagators. The helicity informations are included in the remaining density matrices. When no subscript is included for a given particle with spin in P , it implies that its helicity is summed over. The polarization density matrix $P(i_{\lambda_1 \lambda_1} \bar{i}_{\lambda_2 \lambda_2} \rightarrow t_{\lambda_3 \lambda'_3} \bar{t}_{\lambda_4 \lambda'_4})$ represents the production of the top quark pair via the processes $q\bar{q} \rightarrow t\bar{t}$ or $gg \rightarrow t\bar{t}$. $P(t_{\lambda_3 \lambda'_3} \bar{t}_{\lambda_4 \lambda'_4} \rightarrow b\bar{b}ll\nu\nu)$ is the polarization density matrix for the decay of the top quark pair into b quarks and leptons. This density matrix can be further splitted into the product of the decay density matrix of $t\bar{t}$ into W^+W^- bosons and $b\bar{b}$ pairs, and the decay density matrix of W^+W^- boson pair into a pair of fermions each.

The polarized density matrix of the process $t_{\lambda_3 \lambda'_3} \bar{t}_{\lambda_4 \lambda'_4} \rightarrow b\bar{b}ll\nu\nu$ can be written as

$$P(t_{\lambda_3 \lambda'_3} \bar{t}_{\lambda_4 \lambda'_4} \rightarrow b\bar{b}l^+l^-\nu\bar{\nu}) = \sum_{\lambda_5, \lambda'_5, \lambda_6, \lambda'_6} P(t_{\lambda_3 \lambda'_3} \rightarrow W_{\lambda_5 \lambda'_5}^+ b) P(\bar{t}_{\lambda_4 \lambda'_4} \rightarrow W_{\lambda_6 \lambda'_6}^- \bar{b}) \times |\Pi_W(p_1)|^2 |\Pi_W(p_2)|^2 P(W_{\lambda_5 \lambda'_5}^+ \rightarrow l^+\bar{\nu}) P(W_{\lambda_6 \lambda'_6}^- \rightarrow l^-\nu). \quad (2)$$

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Simple analytic expressions for all the polarization density matrices appeared in the first hand side of above equation can be found in Ref.¹. These expressions can be combined together with the narrow width approximation for the W boson propagators and obtain

$$P(t_{\lambda_3\lambda'_3}\bar{t}_{\lambda_4\lambda'_4} \rightarrow b\bar{b}ll\nu\nu) = 16m_t^2 \left(\frac{e^4 |V_{tb}|^2}{2\sin^4\theta_w} \right)^2 \frac{\pi^2 \delta(p_1^2 - m_W^2) \delta(p_2^2 - m_W^2)}{(M_W \Gamma_W)^2} \\ \times (l_1 \cdot e_1^a)(k_1 \cdot q_1)(l_2 \cdot e_2^b)(k_2 \cdot q_2) \sigma_{a(\lambda_3\lambda'_3)} \otimes \bar{\sigma}_{b(\lambda_4\lambda'_4)}. \quad (3)$$

$$P(t_{\lambda_3\lambda'_3}\bar{t}_{\lambda_4\lambda'_4} \rightarrow b\bar{b}l^+l^-\nu\bar{\nu}) = 16m_t^2 \left(\frac{e^4 |V_{tb}|^2}{2\sin^4\theta_w} \right)^2 \frac{\pi^2 \delta(p_1^2 - m_W^2) \delta(p_2^2 - m_W^2)}{(M_W \Gamma_W)^2} \\ \times (l_1 \cdot e_1^a)(k_1 \cdot q_1)(l_2 \cdot e_2^b)(k_2 \cdot q_2) \sigma_{a(\lambda_3\lambda'_3)} \otimes \bar{\sigma}_{b(\lambda_4\lambda'_4)}. \quad (4)$$

The helicities of final state fermions are summed over. We shall use four vectors k_i , l_i and q_i to represent the momenta of the neutrinos, the charged leptons and the b quarks respectively. The subscript 1 is used to label the momenta related to the decay products of the top quark, while the subscript 2 is used for those related to the decay of the anti-top quark. $\epsilon_{1\mu}(\lambda_5)$ represents the polarization vectors of W^+ with helicity λ_5 , while $\epsilon_{2\nu}(\lambda_6)$ represents those of the W^- with helicity λ_6 . In our notation, the information about the top helicity is carried by the Pauli matrix σ_i and $\sigma_0 (= 1)$ with component $(\lambda_3\lambda'_3)$. For the anti-top, we used the conjugate matrix $\bar{\sigma}_a = \sigma_2\sigma_a\sigma_2$ with component $(\lambda_4\lambda'_4)$. The tensor $e^a(r_i)$, ($a = 0, 1, 2, 3$), were introduced in ref.³ to evaluate the product of fermion wave functions, $u(r, \lambda)\bar{u}(r, \lambda')$, in the amplitude squared for the cross section.

With this, one can then proceed to glue together the density matrix for the production, which has been given in Ref.³, and those for the decays of the top quarks. For the top and anti-top propagators, we shall use also the narrow width approximation. These provide another two delta functions. After that, we can integrate out all the delta functions in the phase space for 6 final state particles and obtain the differential cross section

$$d\sigma_{ii} = \frac{P}{4i_1 \cdot i_2} d_6(T \rightarrow k_1 l_1 q_1 k_2 l_2 q_2) \\ = \frac{4\pi^2 \alpha_s^2}{N} \left(\frac{e^4 |V_{tb}|^2}{2\sin^4\theta_w} \right)^2 (k_1 \cdot q_1)(k_2 \cdot q_2) l_{1\alpha} l_{2\beta} I_{ii}^{\alpha\beta} \frac{d_2(T \rightarrow r_1 r_2)}{i_1 \cdot i_2} \\ \times \frac{d_2(r_1 \rightarrow p_1 q_1) d_2(r_2 \rightarrow p_2 q_2) d_2(p_1 \rightarrow l_1 k_1) d_2(p_2 \rightarrow l_2 k_2)}{(M_W \Gamma_W M_t \Gamma_t)^2}, \quad (5)$$

where $T^\mu = i_1^\mu + i_2^\mu$, with i_1 and i_2 defined as the four-momenta of the initial partons and $d_2(a \rightarrow bc)$ is the phase space for a particle with four-momentum a decaying into particles with four-momenta b and c . N is the number of color and the indices ii designate the initial particles. For the $gg \rightarrow t\bar{t}$ case, we have the result

$$I_{gg}^{\alpha\beta} = \left(x - 1 - \frac{1}{N^2 - 1} \right) \left\{ 2r_1^\alpha r_2^\beta \left[\frac{-1}{x} + \frac{1}{\gamma^2} \left(1 - \frac{x}{2\gamma^2} \right) + 1 - \frac{x}{2\gamma^2} \right] \right. \\ \left. - \frac{1}{2\gamma^2} \left(1 - \frac{x}{\gamma^2} \right) \left(r_1^\alpha T^\beta + T^\alpha r_2^\beta \right) + m_t^2 \left(\frac{-1}{x} + \frac{1}{\gamma^2} \left(1 - \frac{x}{2\gamma^2} \right) \right) g^{\alpha\beta} \right. \\ \left. + \frac{1}{4\gamma^2} \left(1 - \frac{x}{2\gamma^2} \right) \left[T^\alpha T^\beta - R^\alpha R^\beta - \left(\frac{u-t}{s} \right) (R^\alpha T^\beta - T^\alpha R^\beta) \right] \right\} \quad (6)$$

and for the $q\bar{q} \rightarrow t\bar{t}$ case

$$I_{q\bar{q}}^{\alpha\beta} = \left(\frac{N^2 - 1}{2N} \right) \left\{ 2r_1^\alpha r_2^\beta \left[1 - \frac{1}{x} \left(1 - \frac{x}{2\gamma^2} \right) \right] - \frac{1}{2\gamma^2} \left(r_1^\alpha T^\beta + T^\alpha r_2^\beta \right) \right. \\ \left. - \frac{m_t^2}{x} \left(1 - \frac{x}{2\gamma^2} \right) g^{\alpha\beta} + \frac{1}{4\gamma^2} \left[T^\alpha T^\beta - R^\alpha R^\beta - \left(\frac{u-t}{s} \right) (R^\alpha T^\beta - T^\alpha R^\beta) \right] \right\}. \quad (7)$$

The s , t and u are the Mandelstam variable, $R^\mu = i_1^\mu - i_2^\mu$, $\gamma^2 = s/4m_t^2$ and $x = s^2/2(t - m_t^2)(u - m_t^2)$.

To show the spin effect of the quark, we shall compare with the similar calculation in which top quark helicities are averaged over both in the production density matrix and in the decay density matrix independently before we join them together. We shall refer to this case as the "spinless case". In the usual Monte Carlo simulation for the experiments, this is indeed the approximation taken by the program. The cross section for the spinless case can be obtained by substituting I_{gg} and I_{qq} with

$$I_{gg}^{\alpha\beta} = \left(x - 1 - \frac{1}{N^2 - 1}\right) \left\{ r_1^\alpha r_2^\beta \left[1 - \frac{1}{x} + \frac{1}{\gamma^2} \left(1 - \frac{x}{2\gamma^2} \right) \right] \right\} \quad (8)$$

and

$$I_{qq}^{\alpha\beta} = \left(\frac{N^2 - 1}{2N} \right) \left\{ r_1^\alpha r_2^\beta \left(1 - \frac{1}{x} \left(1 - \frac{x}{2\gamma^2} \right) \right) \right\}. \quad (9)$$

For the last two equation, we should emphasize that the cross section without the spin correlation for the top quark is the one that has been used in the literature and by most of the experimental event simulators.

To measure interference effect, we need to find observables that correlate the kinematic variables associated with the particles from the top quark decay and those from the anti-top quark decay. Intuitively, it is not obvious what is the best correlated observable which can probe this interference effect. For the rest of this paper, we shall present our attempts in this direction.

To probe the interference effect, we shall look for observables related to the final decay products, instead of using directly the top momenta which need to be reconstructed. The easiest observables to try are those related to the two charged leptons (electron or muon) from the leptonic decays of two W 's. The advantage is that these particles can be easily observed in a hadron collider. On the other hand, the dilepton decays has lower branching ratios, which reduces the statistics. For this case we had investigated ¹ the effect of interference terms on the distribution of (1) their total energy $E_{l_1} + E_{l_2}$, (2) their total z-momentum $l_{1z} + l_{2z}$, (3) an orthogonal combination $l_{1z} - l_{2z}$, (4) the cosine of the angle between these two charged leptons and (5) the asymmetries A_P to be defined later.

Here we shall simply report the observables that we think are most promising. We shall demonstrate that it is possible to observe such asymmetries at 3σ level with Tevatron II. The lepton correlated asymmetry, A_P , with respect to a plane P passing through the interaction point is defined as follows. Let \vec{l}_1 be the momentum of the charged lepton associated with top decay evaluated in the top rest frame. Similarly, let \vec{l}_2 be that associated with anti-top decay evaluated also in its own rest frame. Then, define $A_P = (N_+ - N_-)/(N_+ + N_-)$ where N_+ is the number of events in which both \vec{l}_1 and \vec{l}_2 lies on the same side of P while N_- is the number of events with both \vec{l}_1 and \vec{l}_2 lying on the opposite side of P . By choosing different P , one can construct different asymmetries. These asymmetries vanish when the effects of both top spin and W spin are ignored (or averaged over) and they remain small even when the W spin effects are included. This property makes them the ideal candidates for the observation of the $t\bar{t}$ spin correlation effects.

In order to measure the lepton correlated asymmetries it is essential to make full reconstruction of the top dilepton events. We assume that the top mass will be well measured in Tevatron Run I and Run II. For dilepton candidate events, we further assume that the missing transverse momentum measured is equal to the sum of the missing transverse momenta of the two neutrinos associated with the dilepton. Contribution to the missing transverse momentum from other neutrinos in the event reduces the efficiency of the reconstruction and lowers the signal-to-noise ratio but does not spoil the observability of the asymmetries we consider here.

The analysis of the productions and the decays of $t\bar{t}$ in hadronic collider with and without spin correlation effect can be found in the literature^{1,6} with various degrees of sophistications. We shall use the simple analytic differential cross sections for $q\bar{q}$ and $gg \rightarrow t\bar{t} \rightarrow bW^+ \bar{b}W^- \rightarrow b\bar{b}\nu_l \bar{\nu}_{l'}$ provided in Ref.¹. Using these formulas, we simulated the $t\bar{t}$ production and decay employing the event generator PYTHIA 5.7. The algorithm of our reconstruction of the top dilepton events has been described in detail in Ref.².

In principle, one can also look for the W correlated asymmetries for the two W 's from $t\bar{t}$ decays in the reconstructed top dilepton, lepton-jet and jet-jet events. The predicted W asymmetries can be easily obtained from Ref.¹. They are no more than a few per cent and about an order of magnitude smaller than some of the lepton correlated asymmetries. The enhancement of the lepton correlated asymmetries over the W correlated asymmetries² is one feature that favors the observation of top spin correlation effects through the dilepton channel.

In hadron colliders, $t\bar{t}$ are produced either by quark-antiquark ($q\bar{q}$) annihilation or by gluon-gluon (gg) fusion. The lepton correlated asymmetries for these two cases typically have opposite sign and with different magnitudes². At Tevatron energy, the number of $q\bar{q}$ produced top events are roughly eight times that of the gg produced events. This ratio decreases with energy and, at LHC energy, the number of gg produced top events are roughly four times that of the $q\bar{q}$ produced events. For the asymmetries we considered, we found that the substantial asymmetries that can be observed at Tevatron become much smaller at the expected LHC collider energy. This is the result of accidental cancellation between the contribution of the $q\bar{q}$ production channel and that of the gg production channel. We have checked that the contribution due to the $q\bar{q}$ channel alone is indeed not so small. This cancellation could be a generic feature which may make LHC unfavorable machine to look for top spin correlation effect. For the same reason, the next linear colliders should be an ideal machine for observing such correlation due to the absence of such cancellation.

As examples, we shall discuss the asymmetries with respect to the following planes defined in the $t\bar{t}$ center of mass frame: (1). $t\bar{t}$ production plane (defines asymmetry A_1); (2). the plane perpendicular to the production plane and contains the top (asymmetry A_2); (3). the plane perpendicular to the two previous planes (asymmetry A_3); (4). the plane normal to the beam direction (asymmetry A_4). These planes are chosen as samples to demonstrate the possibilities. The optimal choice will depend on the details of the detectors and clearly need further study².

The typical trigger for top dilepton events, namely, $p_T \geq 20\text{GeV}$ for leptons, $p_T \geq 10\text{GeV}$ for b -jet, missing transverse energy $E_\perp \geq 25\text{GeV}$ and rapidity $|\eta| \leq 2$, was applied. Afterwards, the reconstruction algorithm we described earlier was carried out. A typical hadron calorimeter energy resolution of $70\%/\sqrt{E}$ was used as b -jet energy smearing and the missing transverse energy was Gaussian smeared with a 15% standard deviation. The effect of including contribution of other neutrinos in an event to the missing transverse energy was investigated². To isolate the effect of the bias originated from reconstruction algorithm to the asymmetries, we also studied the case with the event reconstruction turned on but with the trigger turned off. Similarly we also studied the case with the event reconstruction turned off but with the trigger turned on. The effect of energy smearing is also investigated. In each case, we computed the lepton correlated asymmetries, the neutrino correlated asymmetries and the W correlated asymmetries with respect to the planes described earlier. In Table 1, we present the measured correlated asymmetries A_1, A_2, A_3 and A_4 for the four cases: (I). trigger off, reconstruction off; (II). trigger on, reconstruction off; (III). trigger off, reconstruction on; (IV). trigger on, reconstruction on; (V). trigger on, reconstruction on and with energy smearing included. The corresponding asymmetries when the top spins were "uncorrelated" (that is, their spins are summed over in their productions and averaged over in their decays,) are given in brackets. The average values and the standard deviations of the asymmetries were extracted directly from the simulated data. When both top and W spins were "uncorrelated", as in most standard event generator packages, we verified that all the asymmetries vanish if the trigger and event reconstruction were turned off. The top quark mass is taken to be $m_t = 176\text{ GeV}$ in all numerical analyses. As one can see in the table, the asymmetries measured by charged leptons are generally larger than the asymmetries measured by neutrinos and W bosons. The systematic effects of trigger and reconstruction are quite obvious in the case of neutrino asymmetries as one would expect from the large missing E_\perp cut as well as the contributions from other neutrinos in the event. Due to space, a detailed discussions of the various effects and their origins will be given elsewhere.

From the results of these simulation, we conclude that the lepton correlated asymmetries arising from the $t\bar{t}$ spin correlation can be observed at 3σ level with a few hundred top dilepton events. This is certainly reachable with the projected luminosity of the Tevatron for Run II, with improved detector resolution and acceptance expected for both CDF and D0 detectors upgrades and perhaps with improved

algorithms for identifying the top dilepton events.

There are also plenty of rooms for improvement on our analysis of the correlated asymmetries. Even though trigger and reconstruction show little systematic effects for the lepton asymmetry, they do shift the neutrino and the W asymmetries by non-negligible amounts. A quantitative understanding of these effects may allow us to tune the trigger selection and reconstruction algorithm for the observation of the asymmetries. It is well known that discrete ambiguities exist in general in the reconstruction of top dilepton events. A detailed quantitative study of its effect on the asymmetries could precipitate an improvement on reconstruction algorithm for better efficiency and better signal-to-noise ratio. We have been quite casual in choosing the planes to define the asymmetries and in choosing the combinations of these asymmetries to measure. One may wonder if there is an optimal choice of plane or combinations of planes that can maximize the observability of the correlation effect. One may also wonder if there are choices that will enhance or suppress the contribution of $q\bar{q}$ annihilation channel relative to the gg fusion channel.

Before we conclude, we shall mention two other works^{7,8} which appear in the literature recently. These two papers ignore the off-diagonal terms of the top production density matrix and discuss only the spin correlated asymmetry. In contrary, our work addresses the more general density correlated asymmetry in top production. Only the contributions of the diagonal terms can be described as spin correlated. When off-diagonal terms are included in the analysis, most of the time the top quarks are produced in mix states of helicity(or spin) variable. As a result, our analysis includes effects that cannot be described as spin correlated asymmetry. Of course, for some observables the contributions of the off-diagonal density matrix may happen to be negligible. However it is generally not the case. In fact the optimal case we discovered, the $A_4(l)$ asymmetry in the Table, cannot be described correctly by ignoring the off-diagonal terms. While the spin correlated asymmetry may enjoy simpler intuitive understanding, it, however, does not reflect the full range of possible interference effects emerged in the density matrix correlation in the top pair production.

In conclusion, top spin correlation is certainly one of the most interesting top physics to be uncovered. It can provide a direct observation of the spin 1/2 character of top quark (which we have not been able to do for the lighter quarks) and can potentially test the $V - A$ character of the weak charged current associated with top. We have clearly demonstrated the possibility of observing this effect at Tevatron Run II. The fact that this effect may be even harder for LHC to measure should make the task more important for Tevatron. A detailed account of our study will be presented elsewhere⁵.

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1. D. Chang, S.-C. Lee and P. Turcotte, hep-ph-9508357, to appear in Chinese Journal of Physics (1996).
2. D. Chang, S.-C. Lee and A. Soumarokov, Preprint hep-ph-9512417, to appear in Physical Review Letters.
3. S.C. Lee, Phys. Lett. B **189**, 461, 1987.
4. G.L. Kane, G.A. Ladinsky, C.-P. Yuan, Phys. Rev. D **45**, 124, 1992.
5. D. Chang, C.-H. Chen, S.-C. Lee and A. Soumarokov, in preparation.
6. R. Kleiss and W.J. Stirling, Z. Phys. C **40** 419 (1988); V. Barger, J. Ohnemus and R.J.N. Phillips, Int. J. Mod. Phys. A **4** 617 (1989).
7. G. Mahlon and S. Parkes, hep-ph-9512264, Dec. (1995).
8. T. Stelzer and S. Willenbrock hep-ph-9512292, Dec. (1995).

Table 1: The measured correlated asymmetries $A_i(l)$'s, $A_i(\nu)$'s and $A_i(W)$'s for the four cases: (I). trigger off, reconstruction off; (II). trigger on, reconstruction off; (III). trigger off, reconstruction on; (IV). trigger on, reconstruction on; (V). trigger on, reconstruction on with energy smearing. The corresponding asymmetries when the top spins were “uncorrelated” are given in brackets. All the values are in unit of percentage. The total number of simulated events is 100000.

	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>	<i>V</i>
$A_1(l)$	3.99(−.11)	5.90(.58)	4.16(.68)	6.42(1.94)	5.50(1.14)
$A_2(l)$	−11.78(.41)	−11.18(−.55)	−10.30(−.30)	−7.98(.12)	−7.58(−.06)
$A_3(l)$	−9.65(−.09)	−8.68(−1.34)	−8.00(−1.22)	−6.80(−1.4)	−6.64(−1.26)
$A_4(l)$	−20.34(.18)	−17.64(−.13)	−16.68(−1.86)	−13.72(−1.66)	−13.30(−2.00)
$A_1(\nu)$	0.54(−.68)	9.50(8.86)	5.44(4.70)	12.24(11.80)	9.02(8.48)
$A_2(\nu)$	−1.95(−.06)	.60(1.68)	1.80(2.20)	2.16(2.42)	2.56(2.52)
$A_3(\nu)$	−1.45(−.09)	3.62(4.30)	−1.94(−1.54)	2.64(2.82)	.74(1.22)
$A_4(\nu)$	−3.41(−.37)	−1.84(−.13)	.33(.27)	−.02(.24)	.84(1.02)
$A_1(W)$.79(.16)	3.24(2.32)	1.92(1.34)	3.94(3.18)	3.70(2.92)
$A_2(W)$	−1.98(−.51)	−.48(.33)	−3.34(−.12)	−3.14(.58)	−2.96(.58)
$A_3(W)$	−2.70(−1.46)	.56(.42)	−3.70(−1.44)	−3.64(−1.32)	−4.60(−2.34)
$A_4(W)$	−3.66(−.91)	−3.32(−.04)	−7.12(−1.68)	−6.58(−.72)	−7.57(−1.62)